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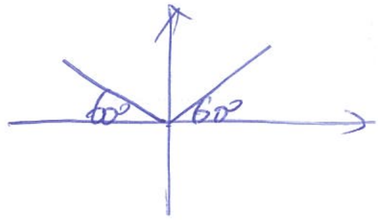
11) $4\sin 7x\cos 2x = \sqrt{3} + 2\sin 9x$. $\quad 0 \le x \le 180°$

$2(\sin 9x + \sin 5x) = \sqrt{3} + 2\sin 9x \quad 0 \le 5x \le 900°$

$2\sin 5x = \sqrt{3}$



$\sin 5x = \frac{\sqrt{3}}{2}$

$5x = 60°, 120°, 420°, 480°, 780°, 840°$.

$x = 12°, 24°, 84°, 96°, 156°, 168°$.

12) $\sin 7x + \sin 3x = 2\sin\left(\frac{7+3}{2}x\right)\cos\left(\frac{7-3}{2}x\right)$

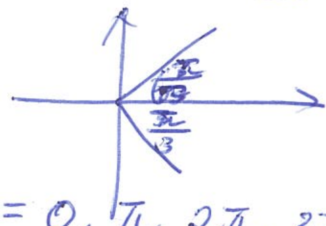
$= 2\sin 5x\cos 2x = \sin 5x$.

I : $2\cos 2x = 1 \quad \cos 2x = \frac{1}{2}$

$0 \le x \le \pi$

$0 \le 2x \le 2\pi$

$0 \le 5x \le 5\pi$



$2x = \frac{\pi}{3}, + \frac{5\pi}{3}$

$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$

II : $\sin 5x = 0 \quad 5x = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$, $x = 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}, \pi$

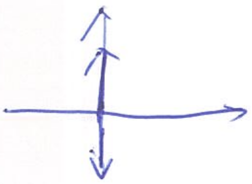
13) $\sin 3x - \sin x = 0$

$2\cos\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right) = 0$

$2\cos 2x\sin x = 0$.

① $\cos 2x = 0$ OR ② $\sin x = 0$

$0 \le 2x \le 720°$. $\quad x = 0°, 180°, 360°$



$2x = 90°, 270°, 450°, 630°$.

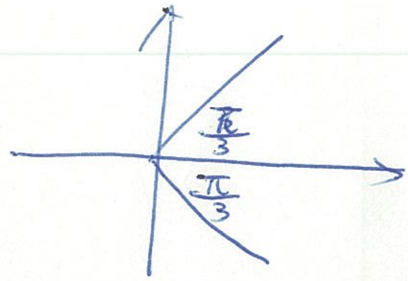
$x = 45°, 135°, 225°, 315°$.

So $x = \{0°, 45°, 135°, 180°, 225°, 315°, 360°\}$.

(14) $\sin 5x \cos 3x = \sin 6x \cos 2x \qquad -\pi \le x \le \pi$

$LHS = \sin 5x \cos 3x = \frac{1}{2}(\sin 8x + \sin 2x) \qquad -2\pi \le x \le 2\pi$

$RHS = \sin 6x \cos 2x = \frac{1}{2}(\sin 8x + \sin 4x)$.



$\sin 2x = \sin 4x = 2\sin 2x \cos 2x$

(I) $\therefore \; 1 = 2\cos 2x \qquad \therefore \cos 2x = \frac{1}{2}$

$2x = \frac{\pi}{3}, \frac{5\pi}{3}, -\frac{\pi}{3}, -\frac{5\pi}{3} \qquad \therefore \underline{x = \frac{\pi}{6}, \frac{5\pi}{6}, -\frac{\pi}{6}, \frac{-5\pi}{6}}$

(II) $\sin 2x = 0. \quad 2x = 0, \pi, 2\pi, -\pi, -2\pi$

$\underline{x = 0, \frac{\pi}{2}, \pi, -\frac{\pi}{2}, -\pi}$.

so $x = \left\{0, \frac{\pi}{6}, \frac{\pi}{2}, \pi, \frac{5\pi}{6}, \frac{-5\pi}{6}, -\frac{\pi}{2}, -\pi, \frac{-\pi}{6}\right\}$.

(15) $\dfrac{\sin A + \sin B}{\cos A + \cos B} = \dfrac{2\sin\left(\frac{A+B}{2}\right)\cancel{\cos\left(\frac{A-B}{2}\right)}}{2\cos\left(\frac{A+B}{2}\right)\cancel{\cos\left(\frac{A-B}{2}\right)}} = \dfrac{\sin\left(\frac{A+B}{2}\right)}{\cos\left(\frac{A+B}{2}\right)} = \tan\left(\frac{A+B}{2}\right)$

(16) $LHS = \sqrt{2}\left(\cos 2x \cos\frac{\pi}{4} + \sin 2x \sin\frac{\pi}{4}\right) - \dfrac{2\sin(5\pi)\cos(2\pi)}{2\sin(5\pi)}$

$= \sqrt{2}\cdot\cos 2x \cdot \frac{\sqrt{2}}{2} + \sqrt{2}\cdot \sin 2x \cdot \frac{\sqrt{2}}{2} - \cos 2\pi$

$= \cos 2x + \sin 2x - \cos 2x = \sin 2x = R.HS$.

(17) $LHS = \frac{1}{2}(\cos 10A + \cos 6A) - \frac{1}{2}(\cos(10A) + \cos 4A)$

$+ \frac{1}{2}(\cos 4A - \cos 6A)$

$= \frac{1}{2}\cos 10A + \frac{1}{2}\cos 6A - \frac{1}{2}\cos 10A - \frac{1}{2}\cos 4A + \frac{1}{2}\cos 4A$

$- \frac{1}{2}\cos 6A = 0 = RHS$.

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18. Prove: $4 \sin 3A \sin 2A \cos A = 1 + \cos 2A - \cos 4A - \cos 6A$

$LHS = 4 \sin 3A \sin 2A \cos A$

$= 4 \left[\sin 3A \sin 2A \right] \cos A$

$= 4 \left[\frac{1}{2} \left(\cos(3A - 2A) - \cos(3A + 2A) \right) \right] \cos A$

$= (2\cos A - 2\cos 5A)(\cos A)$

$\cos 2A = 2\cos^2 A - 1$ $= 2\cos^2 A - 2\cos 5A \cos A$

$= \cos 2A + 1 - 2\left[\frac{1}{2} (\cos 6A + \cos 4A) \right]$

$= \cos 2A + 1 - \cos 6A - \cos 4A$

$= 1 + \cos 2A - \cos 4A - \cos 6A$

$= RHS.$